

3D Equivalent Model to Compute the Electro-Magnetic Behaviour of Twisted Multi-filamentary Superconductors Wires

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In this paper, we investigate new methods to model in 3D, efficiently and simply, twisted multi-filamentary superconductors subjected to an external magnetic field. Several simulation cases, based on the \mathbf{H} -formulation implemented in GetDP, will help derive through appropriate methods, such as changing the reference from where the problem is seen and described to a reference where everything becomes simple to deal with, a simple equivalent 3D problem to solve efficiently and quickly.

Index Terms—High-temperature superconductors, Finite element methods, Multi-filamentary, Maxwell equations.

I. INTRODUCTION

SEVERAL high-temperature superconductivity applications, such as motors design for aircraft propulsion [1] and magnets design for medical imaging [2], use twisted multi-filamentary superconductors wires for their windings.

Both the fine superconducting filaments structure and the twisting process greatly help reduce the AC losses by keeping the filaments as much uncoupled as possible. However, a numerical and precise evaluation of AC losses generated is complex even with the use of robust 3D numerical tools [3].

In addition to the non-linearities arising from the power law [4] and the complex magnetic configuration present in machine, the complex 3D geometry of twisted multi-filamentary superconducting wire will generate a huge number of degrees of freedom.

After a description of the problem, based on the \mathbf{H} -formulation and the numerical approach used, we investigate in this paper several simulation cases. The purpose of those cases is to derive an equivalent simple 3D problem to model quickly and precisely the electro-magnetic behaviour of twisted multi-filamentary superconducting wire subjected to an external magnetic field. All the models involved in this study will use GetDP [5].

II. PROBLEM DEFINITION AND FORMULATION

The studied domain Ω consists of a superconducting sub-domain Ω_s , that includes all the filaments, embedded in a non-superconducting sub-domain $\Omega_{n.s.}$. The non-superconducting sub-domain includes both the resistive matrix and the air domain around the wire.

In order to compute the electro-magnetic behaviour of the studied wire, the \mathbf{H} -formulation will be implemented numerically using the finite element methods. The combination of both Maxwell equations and the linear magnetic constitutive law of the model will give the following variational formulation :

$$\int_{\Omega} \frac{\partial \mathbf{H}}{\partial t} \cdot \boldsymbol{\varphi} d\Omega + \int_{\Omega} \kappa \cdot \text{curl} \mathbf{H} \cdot \text{curl} \boldsymbol{\varphi} d\Omega + I^{\mathcal{B}, \mathcal{D}}, \quad \forall \boldsymbol{\varphi} \in H(\text{curl}; \Omega) \quad (1)$$

with the term $I^{\mathcal{D}} = \int_{\Gamma^{\mathcal{D}}} \boldsymbol{\lambda} \cdot \boldsymbol{\varphi} dA + \int_{\Gamma^{\mathcal{D}}} (\mathbf{H} - \mathbf{H}_a) \cdot \boldsymbol{\lambda}' dA$ where the external magnetic field \mathbf{H}_a is imposed on $\Gamma^{\mathcal{D}} \in \partial\Omega$ as a Dirichlet boundary condition using Lagrange coefficients $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}'$.

Both the unknown magnetic field \mathbf{H} and the basis vector function $\boldsymbol{\varphi}$ are projected along edges elements.

Moreover, the non-linear electrical behaviour of superconductors, characterized by a power law [?], implies a non-linear resistivity $\rho = E_c / J_c \|\mathbf{J} / J_c\|^{n-1}$ in Ω_s . The quantities E_c , J_c and n are the critical electric field, the critical current density and the power law exponent associated with the power law. However, the resistivity ρ will be constant in the resistive matrix and the air domain.

The formulation expressed above will be implemented in the open-source software GetDP.

III. EFFECTS OF A TRANSVERSE MAGNETIC FIELD ON A TWISTED MONO-FILAMENT SUPERCONDUCTING WIRE

A twist pitch of a superconducting wire, made of a twisted superconducting filament embedded in a Niobium matrix, is studied. The air domain, surrounding completely the studied wire, is subjected to an sinusoidal transverse magnetic field.

Finite element methods computations of this problem, using GetDP, gave an unexpected current density distribution inside the wire : three current loops are created as shown in fig.1.

This result, although not intuitively predictable, differs from the expected current distribution of a similar magnetic configuration applied to a straight mono-filament superconducting wire. In such case one current loop will be created inside the filament.

Those differences between a twisted and a straight filament subjected to a similar magnetic configuration led us to the conclusion that the twisted superconducting filament see, in

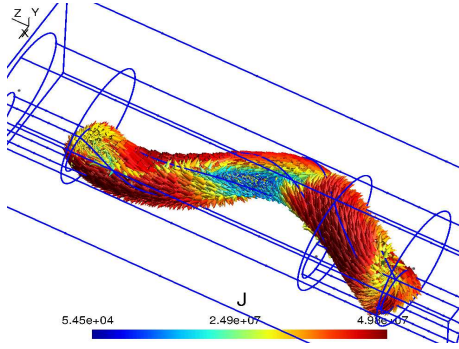


Fig. 1. 3D current density distribution inside the filament at $t = T/4s$

the fixed general Cartesian coordinate system (x, y, z) , a transverse magnetic field with a directional change along its twist pitch. The Cartesian coordinate system moving along the twist trajectory is a Frenet frame (T, N, K) .

Based on the previous results, the Frenet frame can be fixed while the general Cartesian coordinate system becomes the one moving along the twist trajectory. Thus, the twisted filament become a straight one and the transverse magnetic field becomes twisted through the following transformation :

$$\mathbf{H}_{[T,N,K]} = \begin{bmatrix} -a \cdot \sin(\theta) & a \cdot \cos(\theta) & b \\ \cos(\theta) & \sin(\theta) & 0 \\ -b \cdot \sin(\theta) & b \cdot \cos(\theta) & -a \end{bmatrix} \cdot \mathbf{H}_{[x,y,z]} \quad (2)$$

with $a = r/L$, $b = c/L$, $\theta = z/c$, $c = p/2\pi$ and $L^2 = r^2 + c^2$. The radius of the helix formed by the twist transformation is defined by r .

IV. VALIDATION OF THE EXTERNAL MAGNETIC FIELD TWIST TRANSFORMATION ON A STRAIGHT BI-FILAMENTS SUPERCONDUCTING WIRE

In order to validate those previous conclusions, comparisons of the current distribution and AC losses of two simulation cases has been done.

The first case concerns the modeling of the twist pitch p of a twisted bi-filaments superconducting wire subjected to a transverse magnetic field \mathbf{B}_a . The second case is related to the modeling of a straight bi-filaments superconducting wire, of length equivalent to the twist pitch p , subjected to the transformed transverse magnetic field. Parameters of both models are given in the table I.

The computed quantities, shown in fig.2 and 3, are almost equivalent. The results preview an equivalence of both simulation cases.

However, multiple pairs of simulation cases, with different parameters such as the twist pitch, the magnetic field amplitude and the number of filaments, must be performed. Those sets of simulations will help validate completely this equivalence. Those studies will be shown in greater details in the full paper.

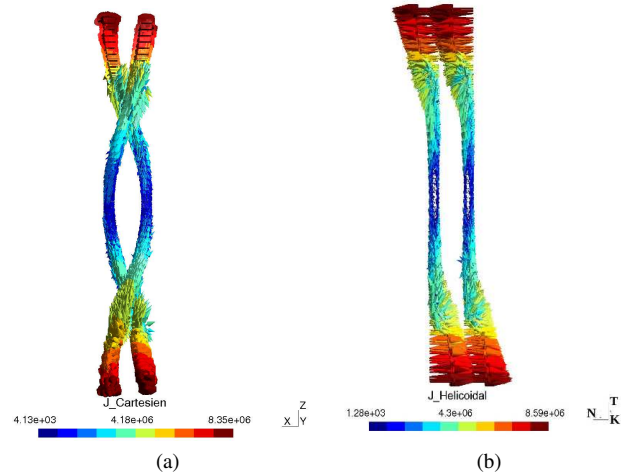


Fig. 2. Current density distribution of (a)-first case simulation and (b)-second case simulation at $t = T/2$ and with $p = 4mm$

TABLE I
MODELS PARAMETERS

Quantity	Value
Filament radius (μm)	60
Matrix radius (mm)	0.4
Critical current density (A/mm^2)	5
Power law index n	10
Matrix conductivity (S/m)	$6 \cdot 10^9$
Magnetic field amplitude B_m (T)	0.1
Frequency f (Hz)	50
Magnetic field norm	$B_m \sin(2\pi f)$

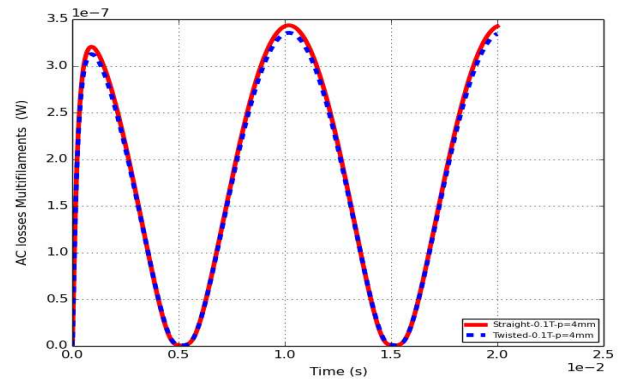


Fig. 3. Comparison of AC losses for a cycle in the superconducting domain for both simulation cases

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